Two regimes of synchronization in unidirectionally coupled semiconductor lasers

I. V. Koryukin[1,2] and Paul Mandel[2]
[1]Institute of Applied Physics, Russian Academy of Science, 46 Ulyanov Street, 603600 Nizhny Novgorod, Russia
[2]Optique Nonlinéaire Théorique, Université Libre de Bruxelles, Campus Plaine, Code Postal 231, 1050 Bruxelles, Belgium
(Received 5 July 2001; published 3 January 2002)

We analyze unidirectionally coupled semiconductor lasers in the feedback/injection scheme to determine their synchronization performance. As the mismatch between the two lasers increases, there is a transition from complete synchronization for identical lasers to time lag synchronization which is only partial. This corresponds to a continuous change of the global minimum that becomes a relative minimum of the synchronization error function and vice versa.

DOI: 10.1103/PhysRevE.65.026201 PACS number(s): 05.45.Xt, 42.55.Px, 42.65.Sf

Recent research on chaotic laser synchronization has been stimulated by potential applications in secure communication. In particular, semiconductor lasers with weak-to-moderate optical feedback attract special interest as a potential device for secure optical communication system due to high-dimensional chaos and gigahertz frequency range of output [1,2]. Three types of synchronization schemes are considered for such laser systems [3]. In the first scheme, both the transmitter and the receiver are similar semiconductor lasers with optical feedback and the coupling being unidirectional. Synchronization in this scheme was reported for different oscillation regimes and for the frequency range from a few kilohertz to a few gigahertz [4,5]. The second scheme is a direct injection scheme in which only the transmitter has an optical feedback loop while the receiver is a solitary semiconductor laser [6]. This system can be viewed as a special case of the previous one with reflectivity of the receiver's feedback mirror tending to zero. Successful chaotic synchronization in this system was demonstrated up to nanosecond time scale [3,7]. The third system consists of two separated semiconductor lasers without additional optical feedback but bidirectionally coupled via the optical field (face-to-face coupling). The chaos synchronization and spontaneous symmetry breaking in this scheme were reported recently [8].

However, high resolution experiments on semiconductor lasers synchronization revealed a pronounced difference between the theoretically predicted and experimentally observed synchronization behaviors. Let τ_c be the transmission time of light from the transmitter laser to the receiver laser and τ_0 the delay time in the transmitter feedback loop. According to the predictions of Ahlers *et al.* [2], chaos synchronization is realized in unidirectionally coupled semiconductor lasers with a time lag $t_d = \tau_c - \tau_0$, for which the equations describing both systems become identical. In the experiments the observed delay time between the two lasers was $t_d = \tau_c$ and was independent of τ_0 [5,7]. The theoretically predicted synchronization regime is the so-called complete synchronization (CS) [9]. It has been studied both for a single-mode [2] and a multimode case [10]. This type of synchronization requires the identity of transmitter and receiver lasers and their oscillation parameters. Full synchronization exists due to the identity of the equations for both systems and the main goal is to determine the stability of this solution. The mismatch in laser parameters between transmitter and receiver leads to a degradation of the synchronization, as was demonstrated in the numerical simulation of direct injection scheme with frequency detuning [11]. Experimentally, the observed behavior is synchronization of the time lag type (LS), which consists in locking the receiver to the output of the transmitter, shifted by some time lag. It has been demonstrated that LS exists in the coupled Rössler systems with nonzero parameter mismatch [13]. On the contrary with CS, not only the stability but also the very existence of a time lag synchronous solution is an open question due to the nonidentity of the equations. In this paper, we focus on the complete and time lag types of synchronization in unidirectionally coupled semiconductor lasers. We show that the degree of CS decreases with mismatch in laser parameters (coupling strength, pump coefficient) and a transition from CS to LS may occur. Our analysis is focused on single-mode lasers because multimode operation does not add qualitatively new features to the present analysis.

Our starting point is a pair of almost identical single-mode semiconductor lasers coupled in a direct injection scheme. The transmitter laser is subjected to coherent optical feedback from an external mirror. The receiver is a similar laser without feedback but in which a fraction of the transmitter output is injected. For single-mode lasers we use the usual Lang-Kobayashi rate equations [14]. After a suitable normalization, the equations become [15]

$$\frac{dE^T}{dt} = (1+i\alpha)F^T E^T + \eta^T E^T(t-\tau_0)e^{-i\Omega^T\tau_0}, \tag{1}$$

$$\frac{dF^T}{dt} = P^T - F^T - (1+2F^T)|E^T|^2, \tag{2}$$

$$\frac{dE^R}{dt} = (1+i\alpha)F^R E^R + \eta^R E^T(t-\tau_c)e^{-i\Omega^T\tau_c}, \tag{3}$$

$$\frac{dF^R}{dt} = P^R - F^R - (1+2F^R)|E^R|^2. \tag{4}$$

Here the indices T,R label the transmitter and receiver variables, E is a slowly varying field amplitude, F is the excess free-carrier density, and α is the linewidth enhancement factor. The reduced time t is measured in units of the photon lifetime τ_p, $T=\tau_s/\tau_p$ is the ratio of the carrier lifetime to the photon lifetime, Ω^T is the solitary transmitter angular frequency. The excess pump current P is proportional to $(I/I_{th})-1$, where I and I_{th} are the current and its value at the solitary laser threshold, respectively. The amount of transmitter feedback is represented by η^T and the injection strength is η^R.

For identical parameters of the transmitter and receiver

$$P^R=P^T\equiv P, \eta^R=\eta^T\equiv\eta, \tag{5}$$

a completely synchronous solution of Eqs. (1)–(4) exists, which is

$$E^R(t)=E^T(t-\Delta t), \quad F^R(t)=F^T(t-\Delta t), \quad \Delta t=\tau_c-\tau_0. \tag{6}$$

Complete synchronization means that both the transmitter and this receiver follow the same trajectory in phase space, with a time lag between systems equal to Δt. The relative position of each system of this trajectory depends on the sign of Δt . For $\tau_c<\tau_0$ time lag is negative and the receiver is in the future state of the transmitter. It has been pointed out [16] that this situation may be considered as anticipated synchronization.

In the following analysis, we choose for simplicity equal delays for the transmitter feedback and the injection to the receiver, $\tau_c=\tau_0$, which leads from Eq. (6) to the completely in-time synchronous solution $E^R(t)=E^T(t)$, $F^R(t)=F^T(t)$, and $t_d\simeq\tau_0$ for time lag synchronization. The fixed parameters for numerical simulations are $\tau_p=1$ ps, $T=10^3,\tau_c=\tau_0=2ns,\alpha=5$. With these parameters and $P=10^{-3},\eta=5\times10^{-3}$, the lasers are in a completely synchronized low-frequency fluctuation regime.

Seeking time lag solution of Eqs. (1)–(4)

$$E^R(t)=E^T(t-\tau),F^R(t)=F^T(t-\tau), \tag{7}$$

with the condition (5), we arrive at the conclusion that the receiver and transmitter fields must be simultaneous solutions of the two different equations

$$\frac{dE^{R,T}}{dt}=(1+i\alpha)F^{R,T}(t)E^{R,T}(t)+\eta E^{R,T}(t)e^{-i\Omega^T\tau_0}, \tag{8}$$

$$\frac{dE^{R,T}}{dt}=(1+i\alpha)F^{R,T}(t)E^{R,T}(t)+\eta E^{R,T}(t-\tau_0)e^{-i\Omega^T\tau_0}. \tag{9}$$

Equations (8) and (9) coincide only if $E^{R,T}(t)=E^{R,T}(t-\tau_0)$. It means that LS of identical lasers may occur only for time periodic solutions, not in the chaotic regime.

The degree of synchronization and the time lag between the lasers can be quantified by the synchronization error function [11], defined as

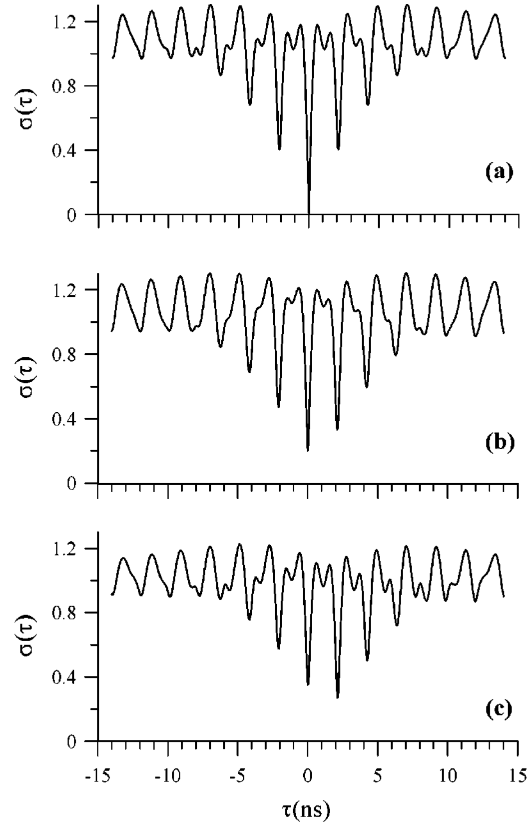

FIG. 1. Synchronization error as a function of the time difference between transmitter and receiver fields for different P^R. The fixed parameters are $T=10^3$, $\tau_c=\tau_0=2$ ns, $\alpha=5$, $\eta^R=\eta^T=5\times10^{-3}$, $P^T=10^{-3}$. (a) Identical lasers, $P^R=10^{-3}$; (b) $P^R=6\times10^{-4}$; (c) $P^R=-3\times10^{-4}$.

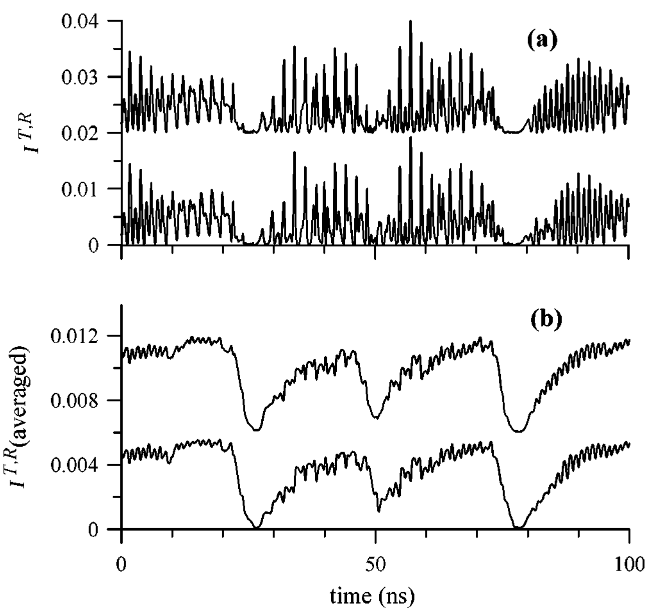

FIG. 2. Laser intensities (a) and averaged laser intensities (b). Transmitter (receiver) output is the upper (lower) trace. Averaging time is 1.5 ns. Parameters as in Fig. 1(b).

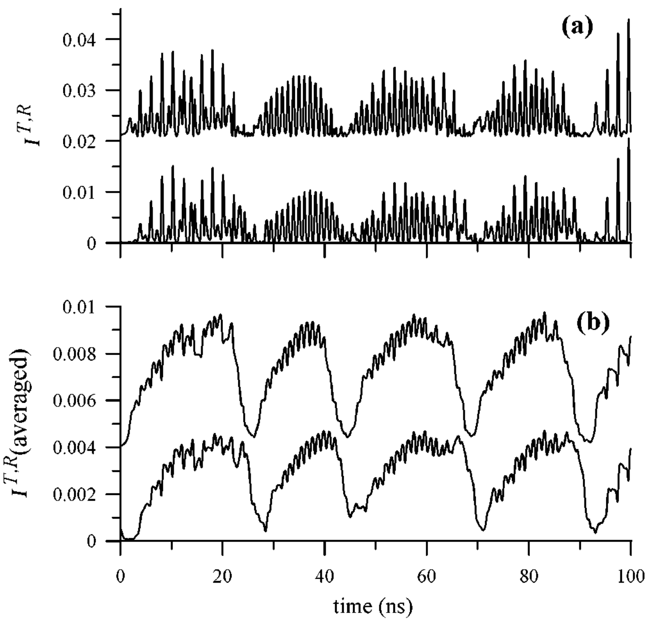

FIG. 3. Laser intensities (a) and averaged laser intensities (b) for the time lag solution. Transmitter (receiver) output is the upper (lower) trace. Averaging time is 1.5 ns. Parameters as in Fig. 1(c).

$$\sigma(\tau)=\frac{\langle|I^R(t+\tau)-I^T(t)|\rangle}{\langle I^T(t)\rangle}, \tag{10}$$

where $I^{R,T}=|E^{R,T}|^2$. The synchronization error as a function of the time shift between transmitter and receiver fields is shown in Fig. 1. For identical lasers this function has a global minimum $\sigma(0)=0$, which corresponds to CS [Fig. 1(a)]. Among other features of this function is the symmetry relative to the change $\tau\to-\tau$ and local minima at $\tau_{\min}\simeq\pm\tau_0$, $\pm 2\tau_0,\ldots,\pm n\tau_0$, separated by τ_0, which are less pronounced as n increases. The mismatch between transmitter and receiver parameters decreases the degree of in-time synchronization [see Fig. 1(b), where the pump parameter of the receiver is decreased but the transmitter pump remains unchanged]. Simultaneously, the symmetry $\tau\to-\tau$ disappears due to an increase (decrease) of the minima for negative (positive) τ. Further increase of the pump parameter mismatch results in changing the global minimum position to the value $\tau_{LS}\simeq\ \tau_0$ [Fig. 1(c)]. This change induces a shift of the whole function $\sigma(\tau)$ [compare Fig. 1(b) and 1(c)], and can be considered as a transition from in-time synchronization (which was CS for identical parameters) to time lag synchronization. More exactly, this second solution is a special case of generalized synchronization [12], which has the form of a time lag in the parameter region we consider. Two points, which confirm this conclusion should be stressed.

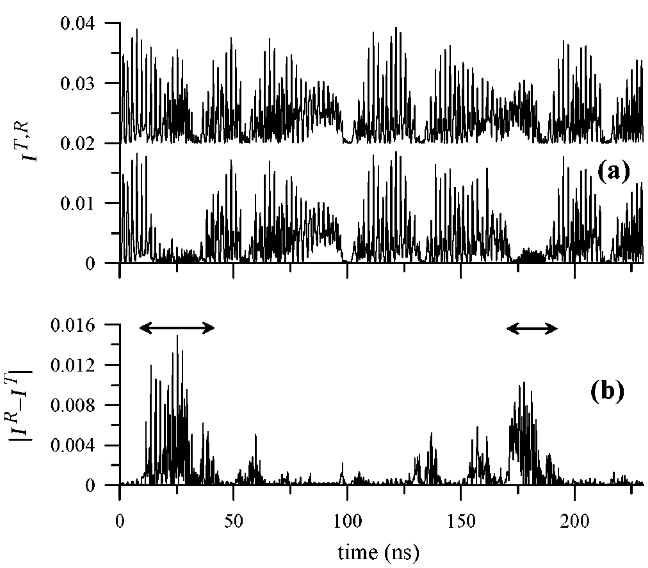

FIG. 4. Intermittent time lag synchronization. Transmitter (upper trace) and receiver (lower trace) intensities (a) and absolute value of their difference (b). The fixed parameters are $P^R=P^T=10^{-3}$, $\eta^T=5\times10^{-3}$, $\eta^R=4.9\times10^{-3}$. Other parameters are as in Fig. 1. Arrows mark the intervals of unsynchronized behavior.

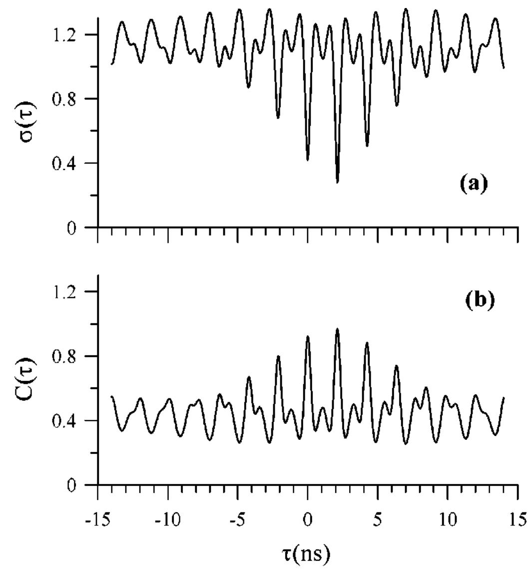

FIG. 5. Synchronization error (a) and correlation function (b) for the mismatch in two parameters $P^T=10^{-3}$, $P^R=0$, $\eta^T=5\times10^{-3}$, $\eta^R=6\times10^{-3}$. Other parameters are as in Fig. 1.

(1) The change of the global minimum position is not due only to a decrease of the synchronization degree at $\tau=0$, but also due to an increase of the synchronization level at $\tau_{LS}\simeq\ \tau_0$.

(2) The position of the global minimum is not strictly at

TABLE I. Minima values of synchronization error function and maxima values of correlation function for single-mode lasers and for the total intensity of multimode lasers. The last column is the ratio of the preceding ones. Parameter values are the same as in Figs. 5 and 6.

	Single mode (SM)	Multimode (MM)	SM/MM
$\sigma(0)$	0.42	0.24	1.75
$\sigma(\tau_{LS})$	0.28	0.17	1.65
$C(0)$	0.92	0.96	0.96
$C(\tau_{LS})$	0.97	0.98	0.99

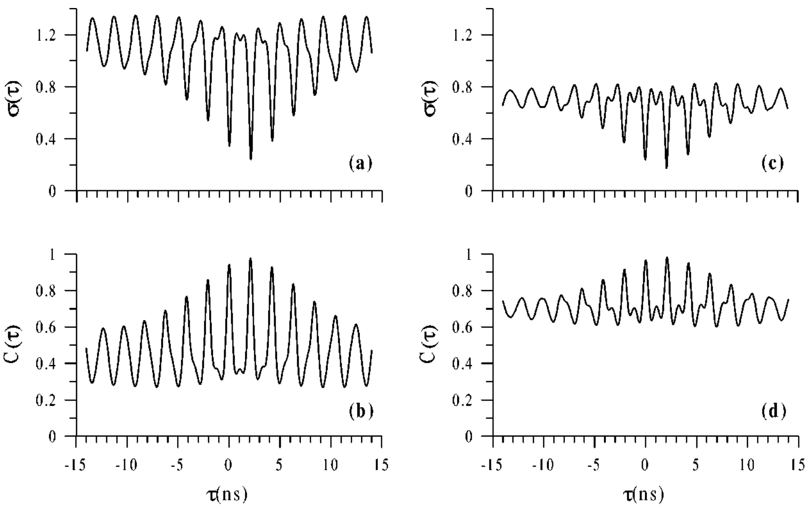

FIG. 6. Synchronization of multimode lasers. Synchronization error and correlation function for the intensity of one mode (a),(b) and for the total intensity (c),(d). The fixed parameters are the same as in Fig. 5 and $\beta^{T,R}=0.22$, $N=3$.

τ_0 ($\tau_{LS}=2.12$ ns, $\tau_0=2$ ns) but slightly depends on the system parameters, as found for LS [13].

Time dependent behaviors of laser intensities and their average values in these two regimes are presented in Figs. 2 and 3 for the parameters of Figs. 1(b) and 1(c). It is clear from Fig. 2 that the transmitter and the receiver have perfectly synchronized wave forms without time lag. Amplitudes of oscillations are slightly different due to the pump parameter mismatch. Short intervals of unsynchronized behavior are only observed just after intensity dropouts. On the contrary, time lag between intensities is clearly identified in Fig. 3, which correspond to LS. This time lag is more pronounced in the average intensities [Fig. 3(b)], but it can also be found very clearly in the high frequency intensity oscillations [Fig. 3(a)] for these values of the parameters.

We now investigate the influence of the mismatch between η^R and η^T on the synchronization behavior. We use the same parameters as before. Increasing η^R (leading to $\eta^R>\eta^T$) results in the same behavior as decreasing P^R. A degradation of the complete synchronization and transition to the time lag synchronization is found. The only difference is quantitative: the decrease of the time lag minimum value $\sigma(\tau_{LS})$ is much smaller than for the pump mismatch. For $\eta^R<\eta^T$, a transition to intermittent time lag synchronization (ILS) instead of stationary time lag synchronization is observed. ILS implies that the two systems are locked in the LS state most of the time, but intermittent bursts of unsynchronized behavior may occur [13,17]. Evolution of the transmitter and receiver intensities together with the modulus of their difference is shown in Fig. 4. Time intervals of unsynchronized behavior are marked by arrows in Fig. 4(b). They appear in addition to short periods of synchronization restoration after intensity dropouts.

So, the mismatch in the parameters of a unidirectionally transmitter/receiver scheme leads to a transition from CS to LS. This effect is reinforced if there is a mismatch in both pump and feedback/injection coefficients. An example of such a situation is presented in Fig. 5 for parameter values $P^T=10^{-3}$, $P^R=0$, $\eta^T=5\times10^{-3}$, $\eta^R=6\times10^{-3}$. Figure 5(a) shows the synchronization error and Fig. 5(b) is the corresponding correlation function, defined as

$$C(\tau)=\frac{\langle I^R(t+\tau)I^T(t)\rangle}{\sqrt{\langle I^R(t)^2\rangle\langle I^T(t)^2\rangle}}. \tag{11}$$

A sharp difference between the main minima of the synchronization error and a sharp difference between the main maxima of the correlation function is observed (Table I).

In conclusion, we have investigated numerically two regimes of chaos synchronization, namely, complete synchronization and time lag synchronization, in unidirectionally coupled semiconductor lasers in a feedback/injection configuration. CS is possible only for identical transmitter and receiver lasers and its degree decreases with parameter mismatch. On the contrary, the time lag regime cannot be fully synchronized for identical lasers. LS requires time periodic solutions and is impossible for chaotic transmitter output. However, the degree of time lag synchronization may increase with the parameter mismatch and a transition from CS to LS may occur. We have found that the synchronization error, as a function of time shift between transmitter and receiver signal, is a useful tool for investigating the synchronization behavior. Furthermore, we have verified that the existence of two synchronization regimes and the transition between them with increasing laser mismatch is a common feature both for single-mode and multimode semiconductor lasers.

A direct integration of the multimode equations of Ref. [18] for three modes and otherwise the same parameters as in this paper does not reveal any qualitative change. Figure 6 displays a sample of these results. We have included in Table I the multimode results for comparison.

It is a pleasure to acknowledge fruitful discussions with E.A. Viktorov. This research was supported by the Belgian Office for Scientific Technical and Cultural Affairs, the Fonds National de la Recherche Scientifique, and the Interuniversity Attraction Pole program of the Belgian government.

[1] I. Fischer, G.H.M. van Tartwijk, A.M. Levine, W. Elsasser, E. Gobel, and D. Lenstra, Phys. Rev. Lett. **76**, 220 (1996).
[2] V. Ahlers, U. Parlitz, and W. Lauterborn, Phys. Rev. E **58**, 7208 (1998).
[3] Y. Takiguchi, H. Fujino, and J. Ohtsubo, Opt. Lett. **24**, 1570 (1999).
[4] S. Sivaprakasam and K.A. Shore, Opt. Lett. **24**, 466 (1999); IEEE J. Quantum Electron **36**, 35 (2000).
[5] H. Fujino and J. Ohtsubo, Opt. Lett. **25**, 625 (2000).
[6] J.K. White and J.V. Moloney, Phys. Rev. A **59**, 2422 (1999).
[7] I. Fischer, Y. Liu, and P. Davis, Phys. Rev. A **62**, 011801(R) (2000).
[8] T. Heil, I. Fisher, W. Elsasser, J. Mulet, and C. R. Mirasso, Phys. Rev. Lett. **86**, 795 (2001).
[9] V.S. Afraimovich, N.N. Verichev, and M.I. Rabinovich, Radiophys. Quantum Electron. **29**, 795 (1986); L.M. Pecora and T.L. Carroll, Phys. Rev. Lett. **64**, 821 (1990).
[10] E.A. Viktorov and P. Mandel, Phys. Rev. A **65**, 015801 (2002).
[11] Y. Liu, H.F. Chen, J.M. Liu, P. Davis, and T. Aida, Phys. Rev. A **63**, 031802(R) (2001).
[12] N.F. Rulkov, M.M. Sushchik, L.S. Tsimring, and H.D.I. Abarbanel, Phys. Rev. E **51**, 980 (1995).
[13] M.G. Rosenblum, A.S. Pikovsky, and J. Kurths, Phys. Rev. Lett. **78**, 4193 (1997).
[14] R. Lang and K. Kobayashi, IEEE J. Quantum Electron. **QE-16**, 347 (1980).
[15] P.M. Alsing, V. Kovanis, A. Gavrielides, and T. Erneux, Phys. Rev. A **53**, 4429 (1996).
[16] C. Masoller, Phys. Rev. Lett. **86**, 2782 (2001).
[17] S. Boccaletti and D.L. Valladares, Phys. Rev. E **62**, 7497 (2000).
[18] E.A. Viktorov and P. Mandel, Phys. Rev. Lett. **85**, 3157 (2000); T.W. Carr, D. Pieroux, and P. Mandel, Phys. Rev. A **63**, 033817 (2001).